# **Tree Terminology**

## **Tree Definition**

A **tree** can be defined in several ways.

One natural way to define a tree is **recursively**.

We can say ***T*** is a tree if either

***T*** has no nodes (is empty)

or

***T*** is of the form

Diagram

Description automatically generated

where we have a top node called **root**

where *T*1, *T*2, . . . , *Tk* are trees

From the recursive definition, we find that a tree is a collection of

***N* nodes**, one of which is the root

and

***N* − 1 edges**

That there are *N* − 1 edges follows from the fact that each edge connects some node to its parent, and every node except the root has one parent.

## **Tree Hierarchies**

All trees are **hierarchical** in nature.

“**Hierarchical**” means that a “**parent-child**” relationship exists between the nodes in the tree.

**Parent-Child**

Node ***n***is a **parent** of node ***m*** if

* An **edge** exists between them
* Node ***n***is **above** node ***m***in the tree

Conversely, we can say that ***m*** is a **child** of node ***n*** if

* An **edge** exists between them
* Node ***m*** is **below** node ***n*** in the tree

Each node in a tree has at most one parent.

One node—called the **root** of the tree—has no parent.

Each node may have an **arbitrary number of children**, possibly zero.

Nodes with no children are known as **leaves**.

**Ancestor-Descendant**

The **parent-child** relationship between nodes is generalized to an **ancestor-descendant** relationship.

Chart, line chart

Description automatically generated

In Figure 15-1a

* *A* is an ancestor of *D*
* *D* is a descendant of *A*

Not all nodes are related by the ancestor or descendant relationship:

*B* and *C,* for instance, do not have a descendant relationship, but are **siblings**

Nodes with the same parent are **siblings**.

The leftmost child is called the **oldest child**, or **first child**.

However, the **root** of any tree is an **ancestor of every node** in that tree**.**

A **subtree** in a tree is any node in the tree together with all of its descendants.

A **subtree of a node *n***is a subtree rooted at a child of ***n***.

For example, Figure 15-1b shows a subtree of the tree in Figure 15-1a.

This subtree has *B* as its root and is a subtree of the node *A*.

Practice:

Shape

Description automatically generated

**root:** *A*

**Siblings**: *B, C, D, E, F, G* are siblings

*H* has no siblings

*I, J* are siblings

*K* *L*, *M* are siblings

*N* has no siblings

*P, Q* are siblings

**Leaves**: *B*, *C*, *H*, *I*, *P*, *Q*, *K*, *L*, *M*, and *N*.

**Parent of Node *F***: *A*

**Children of Node *F***: *K*, *L*, and *M*

**Parent of Node *J***: *E*

**Children of Node *J***: *P* and *Q*

## **Path, Length, Depth, and Height**

A **path** from node *n*1 to *nk* is defined as

A sequence of nodes *n*1,*n*2,...,*nk*

such that

*ni* is the parent of *ni*+1 for 1 ≤ *i* < *k*

Notice that in a tree there is exactly **one path** **from the root to each node**.

If there is a path from *n*1 to *n*2, then

*n*1 is an **ancestor** of *n*2

and

*n*2 is a **descendant** of *n*1.

If *n*1 != *n*2

*n*1 is a **proper ancestor** of *n*2

and

*n*2 is a **proper descendant** of *n*1.

The **length** of this path is the **number of edges on the path**, namely *k* − 1.

There is a path of length zero from every node to itself.

For any node *ni*, the **depth** of *ni* is the length of the unique path from the **root to *ni***.

Thus, the root is at depth 0.

The **height** of *ni* is the length of the longest path from ***ni* to a leaf**.

The height of a tree is equal to the height of the root.

All leaves are at height 0.

Practice:

Shape

Description automatically generated

**Node *F***

Depth 1

Height 1

**Node *E***

Depth 1

Height 2

**Height of the tree**

The height of the tree is equal to the height of the root.

The height of node A is 3, which means that the height of the tree is 3.

**Depth of the tree**

The depth of a tree is equal to the depth of the deepest leaf; this is always equal to the height of the tree.

Therefore, the depth of the tree is also 3.